LETTER TO THE EDITOR

p-Branes from Generalized Yang–Mills Theory

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Abstract. We consider the reduced, quenched version of a generalized Yang–Mills action in $4k$-dimensional spacetime. This is a new kind of matrix theory which is mapped through the Weyl–Wigner–Moyal correspondence into a field theory over a non-commutative phase space. We show that the "classical" limit of this field theory is encoded into the effective action of an open, $(4k-1)$-dimensional, bulk brane enclosed by a dynamical, Chern–Simons type, $(4k-2)$-dimensional, boundary brane. The bulk action is a pure volume term, while the boundary action carries all the dynamical degrees of freedom.

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The relation between gauge theories and relativistic extended objects is one of the most intriguing open problems currently under investigation in high energy theoretical physics. Gauge symmetry is the inspiring principle underlying unification of fundamental forces at the quantum level, gravity not included. A really unified theory, including a consistent quantum theory of gravitational phenomena as well, forces the introduction of relativistic extended objects as the basic building blocks of matter, space and time. If correct, this picture must be able to account for the low energy role of gauge symmetry. The presence of massless vector excitations, carrying Chan–Paton indices in the massless sector of the open string spectrum, is a first step towards the answer of this problem in a perturbative framework. The recent proposals for a non-perturbative formulation of string theory in terms of matrices and D-branes [1] provides further clues in favor of the strings/gauge fields. The problem is equally difficult to deal from the low energy viewpoint, involving non-perturbative aspects of gauge theory. Looking for extended excitations in the spectrum of Abelian gauge theories is a problem dating back to the seminal Dirac's work about strings and monopoles [2]. Recent generalization to higher rank gauge fields has been given in [3], [4], [5]. In the non-Abelian case the problem is even more difficult because of the interplay with confinement [6]. Thus, it can be dealt within some appropriate approximation scheme. Because of the the large value of the gauge coupling constant standard perturbation theory is not available and different computational techniques have to be adopted. One of the most successful is the large-N expansion, where N refers to the number of colors [7]. To match Yang–Mills theory and matrix string theory further approximations are available, i.e. "quenching" and "reduction". The original $SU(N)$ Yang–Mills field is replaced by the same field at a single point [8], say $x^\mu = 0$ (for a recent review see [9]) and represented by a unitary $N\times N$ matrix $\boldsymbol{A}_\mu{}^i_j$. Partial derivative operators are replaced by commutators with a fixed diagonal matrix $\boldsymbol{p}_\mu{}^i_j$, playing the role of translation generator and called the *quenched momentum* [10]. Accordingly, the covariant derivative becomes $\mathrm{i}\boldsymbol{D}_\mu = \left[\boldsymbol{p}_\mu + \boldsymbol{A}_\mu, \ldots\right]$. Thus, the reduced, quenched, Yang–Mills field strength is

$$\boldsymbol{F}_{\mu\nu}{}^i_j \equiv [\mathrm{i}\boldsymbol{D}_\mu, \mathrm{i}\boldsymbol{D}_\nu]^i_j$$

It has been shown in [11] that the dynamics of reduced, quenched, Yang–Mills theory can be formulated in the large-N limit in terms of pure string dynamics. In a recent paper we have generalized this result to include bags and membranes in the spectrum of 4-dimensional Yang–Mills theory [12]. In this note, we shall look for higher dimensional objects fitting into generalized Yang–Mills theories, in more than 4-dimensional spacetime.

Yang–Mills theory admits in $D = 4k$, $k = 1, 2, \ldots$ dimensions a generalization preserving both the canonical dimension of the gauge field, i.e. $[\boldsymbol{A}_\mu] = (\mathrm{length})^{-1}$, and of the coupling constant $[g_{\mathrm{YM}}] = 1$. The action we shall use is of the form introduced in [13]

$$S^{\mathrm{GYM}} = -\frac{1}{2(2k)!g^2_{\mathrm{YM}}} \int d^{4k}x \mathrm{Tr} \boldsymbol{F}_{[\mu_1\mu_2} \cdot \ldots \cdot \boldsymbol{F}_{\mu_{2k-1}\mu_{2k}]} \boldsymbol{F}^{[\mu_1\mu_2} \cdot \ldots \cdot \boldsymbol{F}^{\mu_{2k-1}\mu_{2k}]}, \tag{1}$$

where $\boldsymbol{F}_{\mu_1\mu_2} \equiv \partial_{[\mu_1}\boldsymbol{A}_{\mu_2]} - [\boldsymbol{A}_{\mu_1}, \boldsymbol{A}_{\mu_2}]$, and the trace operation is over internal indices.The reason why this particular generalization of the Yang–Mills action is expected to be of specific interest can be traced back to the original paper where it was introduced [13]. The problem is that the Polyakov action is conformally invariant for a string but does not preserve such a fundamental symmetry for higher dimensional objects. The purpose of the authors in [13] was to look for *conformally invariant* σ–models in $2n$ dimensional spacetime generalizing the Polyakov action. They introduced a generalized Yang–Mills gauge theory fulfilling the above requirements and reproducing the action (1) when $2n = 4k$. In a successive paper the same authors identified the instanton solutions of their generalized Yang–Mills model as "n–branes [14]. This result was extended also to non-compact non-linear σ-models on Anti DeSitter spacetime background in [15]. From our vantage point, we know that the action (1) with $k = 1$ admits both string and bag solutions in the large–N limit [12]. Thus, we are confident to find $4k-1$–branes in the large–N spectrum of (1). This type of objects have a "trivial" bulk dynamics since they are spacetime volume filling solutions carrying no transverse degree of freedom. On the other hand, a $4k-1$–brane embedded into a $4k$–dimensional spacetime must be open by definition, and can admit a non–trivial $4k-2$–brane as its *dynamical boundary.* Thus, we supplement the original Dolan Tchrakian action with a new topological term extending to $4k$ dimension the usual Yang–Mills θ-term:

$$S^\theta = -\frac{\theta g^2_{\rm YM}}{4\pi^2 4^k}\epsilon^{\mu_1\mu_2\ldots\mu_{4k-1}\mu_{4k}}\int d^{4k}x \,{\rm Tr}\boldsymbol{F}_{[\mu_1\mu_2}\cdot\ldots\cdot\boldsymbol{F}_{\mu_{4k-1}\mu_{4k}]}. \tag{2}$$

The main purpose of this note is to establish a correspondence between the action $S^{\rm GYM} + S^\theta$ and some appropriate brane action. As a first step towards this result we turn the gauge field actions $S^{\rm GYM}$ and S^θ into matrix action through quenching and reduction:

$$S^{\rm GYM} + S^\theta \Longrightarrow S^{\rm q,GYM}_{\rm red.} + S^{q,\theta}_{\rm red.}$$

$$\begin{aligned} S^{\rm q,GYM}_{\rm red.} &= -\frac{N}{2(2k)!g^2_{\rm YM}}\left(\frac{2\pi}{a}\right)^{4k}{\rm Tr}\left[\boldsymbol{D}_{[\mu_1}, \boldsymbol{D}_{\mu_2}\right]\cdot\ldots\cdot\left[\boldsymbol{D}_{\mu_{2k-1}}, \boldsymbol{D}_{\mu_{2k}]}\right] \\ &\quad\times\left[\boldsymbol{D}^{[\mu_1}, \boldsymbol{D}^{\mu_2}\right]\cdot\ldots\cdot\left[\boldsymbol{D}^{\mu_{2k-1}}, \boldsymbol{D}^{\mu_{2k}]}\right] \end{aligned} \tag{3}$$

$$S^{q,\theta}_{\rm red.} = -\frac{\theta g^2_{\rm YM}}{4\pi^2 4^k}\left(\frac{2\pi}{a}\right)^{4k}\epsilon^{\mu_1\mu_2\ldots\mu_{4k-1}\mu_{4k}}{\rm Tr}\left[\boldsymbol{D}_{[\mu_1}, \boldsymbol{D}_{\mu_2}\right]\cdot\ldots\cdot\left[\boldsymbol{D}_{\mu_{4k-1}}, \boldsymbol{D}_{\mu_{4k}]}\right]. \tag{4}$$

One of the most effective way to quantize a theory where the dynamical variables are represented by unitary operators is provided by the Wigner–Weyl–Moyal approach [16]. A by–product of the Wigner–Weyl–Moyal quantization of a Yang–Mills matrix theory is that the "classical limit $\hbar \to 0$" is just the same as the large-N limit. Once applied to our problem the Wigner–Weyl–Moyal quantization scheme allows us to write the unitary matrix $\boldsymbol{D}_{\mu}{}^i_j$ in terms of $2n$ independent matrices $\boldsymbol{p}_i$, $\boldsymbol{q}_j$, $i, j = 1, \ldots, n$, $0 \le n \le 4k$ [17],

$$\boldsymbol{D}_\mu \equiv \frac{1}{(2\pi)^D}\int d^np\, d^nq \mathcal{A}_\mu(q,p)\exp\left({\rm i}q^i\boldsymbol{p}_i + {\rm i}p^j\boldsymbol{q}_j\right), \tag{5}$$

where the operators $\boldsymbol{p}_i$, $\boldsymbol{q}^j$ satisfy the Heisenberg algebra

$$\left[\boldsymbol{p}_i, \boldsymbol{q}_j\right] = -{\rm i}\hbar\delta_{ij}$$

and (q^i, p^j) play the role of coordinates in Fourier dual space. $\hbar$ is the *deformation parameter*, which for historical reason is often represented by the same symbol as the Planck constant.

The basic idea under this approach is to identify the Fourier space as the dual of a $2n$-dimensional world manifold of a $p = 2n - 1$ brane. Consistency requires that the dimension of the world surface swept by the brane evolution at most matches the dimension of the target spacetime, and never exceeds it, i.e. $2n \leq 4k$.

The case $n = 1$, $k = 1$, describing string sector of large-N QCD, deserved an in depth investigation [11], [18]; the case $n = 2$, $k = 1$ has been considered in [12]. In this letter, we shall discuss the more general case, $n = 2k$ and show that it contains a $4k - 1$ open brane enclosed by a dynamical boundary.

By inverting (5) one gets

$$\mathcal{A}_\mu(q,p) = \frac{1}{N}\,\mathrm{Tr}_{\mathcal{H}}\left[\boldsymbol{D}_\mu \exp\left(-\mathrm{i}\boldsymbol{p}_i q^i - \mathrm{i}\boldsymbol{q}_j p^j\right)\right], \tag{6}$$

where $\mathrm{Tr}_{\mathcal{H}}$ means the sum over diagonal elements with respect an orthonormal basis in the Hilbert space $\mathcal{H}$ of square integrable functions on R^{4k}. By Fourier anti–transforming (6) one get the Weyl symbol $\mathcal{A}_\mu(x,y)$ of the operator $\boldsymbol{D}_\mu$:

$$\mathcal{A}_\mu(x,y) = \int d^n q\, d^n p\, \mathcal{A}_\mu(q,p) \exp\left(iq_i x^i + ip_j y^j\right).$$

The above procedure turns the product of two matrices $\boldsymbol{U}$ and $\boldsymbol{V}$ into the Moyal, or $*$-product, of their associated symbols

$$\boldsymbol{U}\boldsymbol{V} \longleftrightarrow \mathcal{U}(\sigma) * \mathcal{V}(\sigma) \equiv \exp\left[i\frac{\hbar}{2}\omega^{ab}\frac{\partial^2}{\partial\sigma^a\partial\xi^b}\right]\mathcal{U}(\sigma)\mathcal{V}(\xi)|_{\sigma=\xi}$$
$$\sigma^a \equiv \left(x^k, y^l\right),$$

where ω^{ab} is the symplectic form defined over the dual phase space (x, y). The introduction of the non-commutative $*$-product allows to express the commutator between two matrices $\boldsymbol{U}$, $\boldsymbol{V}$ as the *Moyal Bracket* between their corresponding symbols $\mathcal{U}(x,y)$, $\mathcal{V}(x,y)$

$$\frac{1}{\hbar}\left[\boldsymbol{U}, \boldsymbol{V}\right] \longleftrightarrow \{\mathcal{U}, \mathcal{V}\}_{\mathrm{MB}} \equiv \frac{1}{\hbar}\left(\mathcal{U} * \mathcal{V} - \mathcal{V} * \mathcal{U}\right) \equiv \omega^{ij}\,\partial_i\mathcal{U} \circ \partial_j\,\mathcal{V},$$

where we introduced the $\circ$-product which corresponds to the "even" part of the of the $*$-product [19]. Once each operator is replaced by its own Weyl symbol, the trace operation in Hilbert space turns into an integration over a $2D$-dimensional, non-commutative manifold, because of the ubiquitous presence of the $*$ product [20]:

$$\frac{(2\pi)^4}{N^3}\mathrm{Tr}_{\mathcal{H}} \longmapsto \int d^n x\, d^n y \equiv \int d^{2n}\sigma.$$

The last step of the mapping between matrix theory into a field model is carried out through the identification of the "*deformation parameter*" $\hbar$ with the inverse of N:

$$\text{“}\hbar\text{''} \equiv \frac{2\pi}{N}.$$

Thus, the large-N limit of the $SU(N)$ matrix theory, where the $\boldsymbol{A}_\mu$ quantum fluctuations freeze, corresponds to the quantum mechanical classical limit, $\hbar \to 0$, of the WWM corresponding field theory. From now on, we shall refer to the "classical limit" without distinguishing between the large-N or small-$\hbar$. In the classical limit the Moyal bracket reproduces the Poisson bracket:

$$\mathcal{F}_{\mu\nu} \equiv \{\mathcal{A}_\mu, \mathcal{A}_\nu\}_{\mathrm{MB}} \overset{\hbar\to 0}{\longmapsto} \mathcal{F}^{\infty}_{\mu\nu} \equiv \{\mathcal{A}_\mu, \mathcal{A}_\nu\}_{\mathrm{PB}}\,.$$

The above formulae are all we need to map the matrix actions (3) and (4) into their Weyl symbols:

$$\begin{aligned}
W^{\mathrm{GYM}} = -&\frac{1}{2g^2_{\mathrm{YM}}(2k)!}\left(\frac{2\pi}{a}\right)^{4k}\left(\frac{2\pi}{N}\right)^{2k-4} \\
&\times\int_\Sigma d^{2n}\sigma \mathcal{F}_{[\mu_1\mu_2} * \ldots * \mathcal{F}_{\mu_{2k-1}\mu_{2k}]} * \mathcal{F}^{[\mu_1\mu_2} * \ldots * \mathcal{F}^{\mu_{2k-1}\mu_{2k}]} \\
=& \frac{1}{2g^2_{\mathrm{YM}}(2k)!}\left(\frac{2\pi}{a}\right)^{4k}\left(\frac{2\pi}{N}\right)^{2k-4} \\
&\times\int_\Sigma d^{2n}\sigma \left\{\mathcal{A}_{[\mu_1}, \mathcal{A}_{\mu_2}\right\}_{\mathrm{MB}} * \ldots * \left\{\mathcal{A}_{\mu_{2k-1}}, \mathcal{A}_{\mu_{2k}]}\right\}_{\mathrm{MB}} \\
&* \left\{\mathcal{A}^{[\mu_1}, \mathcal{A}^{\mu_2}\right\}_{\mathrm{MB}} * \ldots * \left\{\mathcal{A}^{\mu_{2k-1}}, \mathcal{A}^{\mu_{2k}]}\right\}_{\mathrm{MB}}\,, \qquad (7)
\end{aligned}$$

$$\begin{aligned}
W^{\theta} = -&\frac{\theta g^2_{\mathrm{YM}}}{4\pi^2 4^k}\left(\frac{2\pi}{a}\right)^{4k}\left(\frac{2\pi}{N}\right)^{2k-4}\epsilon^{\mu_1\mu_2\ldots\mu_{4k-1}\mu_{4k}}\int_{\partial\Sigma} d^{2n}\sigma \mathcal{F}_{[\mu_1\mu_2} * \ldots * \mathcal{F}_{\mu_{4k-1}\mu_{4k}]} \\
=& -\frac{\theta g^2_{\mathrm{YM}}}{4\pi^2 4^k}\left(\frac{2\pi}{a}\right)^{4k}\left(\frac{2\pi}{N}\right)^{2k-4} \\
&\times\epsilon^{\mu_1\mu_2\ldots\mu_{4k-1}\mu_{4k}}\int_{\partial\Sigma} d^{2n}\sigma \left\{\mathcal{A}_{[\mu_1}, \mathcal{A}_{\mu_2}\right\}_{\mathrm{MB}} * \ldots * \left\{\mathcal{A}_{\mu_{4k-1}}, \mathcal{A}_{\mu_{4k}]}\right\}_{\mathrm{MB}}\,. \qquad (8)
\end{aligned}$$

To perform the classical limit of (7) and (8) let us rescale the field $\mathcal{A}_\mu$ as

$$\left(\frac{2\pi}{N}\right)^{\frac{2k-4}{4k}}\mathcal{A}_\mu \longrightarrow X_\mu,$$

and replace the $*$-product with the ordinary, commutative, product between functions. Thus, we obtain

$$\begin{aligned}
W^{\mathrm{GYM}}_\infty = -&\frac{1}{2g^2_{\mathrm{YM}}(2k)!}\left(\frac{2\pi}{a}\right)^{4k}\int_\Sigma d^{4k}\sigma\, \partial_{[\,m_1}X_{\mu_1}\partial_{m_2}X_{\mu_2}\cdot\ldots\cdot\partial_{m_{2k-1}}X_{\mu_{2k-1}}\partial_{m_{2k}}X_{\mu_{2k}\,]} \\
&\times\partial^{[\,a_1}X^{\mu_1}\partial^{a_2}X^{\mu_2}\cdot\ldots\cdot\partial^{a_{2k-1}}X^{\mu_{2k-1}}\partial^{a_{2k}\,]}X^{\mu_{2k}} \\
=& -\frac{1}{2g^2_{\mathrm{YM}}(2k)!}\left(\frac{2\pi}{a}\right)^{4k}\int_\Sigma d^{4k}\sigma\partial_{[m_1}X_{\mu_1}\partial_{m_2}X_{\mu_2}\cdot\ldots\cdot\partial_{m_{2k-1}}X_{\mu_{2k-1}}\partial_{m_{2k}]}X_{\mu_{2k}} \\
&\times\partial^{[\,m_1}X^{\mu_1}\partial^{m_2}X^{\mu_2}\cdot\ldots\cdot\partial^{m_{2k-1}}X^{\mu_{2k-1}}\partial^{m_{2k}\,]}X^{\mu_{2k}} \qquad (9)
\end{aligned}$$

and

$$\begin{aligned}
W^{\theta}_\infty = -&\frac{\theta g^2_{\mathrm{YM}}}{4\pi^2 4^k}\left(\frac{2\pi}{a}\right)^{4k}\epsilon^{\mu_1\mu_2\ldots\mu_{4k-1}\mu_{4k}}\int_{\partial\Sigma} d^{4k-1}s\,\epsilon^{m_2\ldots m_{4k}}X_{\mu_1}\partial_{m_2}X_{\mu_2}\cdot\ldots\cdot\partial_{m_{4k-1}}X_{\mu_{4k}} \\
=& -\frac{\theta g^2_{\mathrm{YM}}}{4\pi^2 4^k}\left(\frac{2\pi}{a}\right)^{4k}\epsilon^{\mu_1\mu_2\ldots\mu_{4k-1}\mu_{4k}}\int_{\partial\Sigma} d^{4k-1}s\,X_{\mu_1}\left\{X_{\mu_2},\ldots,X_{\mu_{4k-1}}\right\}_{\mathrm{NPB}}\,. \qquad (10)
\end{aligned}$$

The two actions (9) and (10) are the main results of this note and the discussion of their physical meaning will end this letter.
W^{θ}_{∞} is the action for the *Chern–Simons* $(4k-2)$-brane, investigated in [21]. This kind of p-brane is a dynamical object with very interesting properties following from its topological origin [22], e.g. the presence of the generalized Nambu–Poisson bracket $\left\{X_{\mu_2}, \ldots, X_{\mu_{4k-1}}\right\}_{\text{NPB}}$ which suggests a new formulation of both classical and quantum mechanics for this kind of object [23].
To identify the kind of physical object described by W^{GYM}_{∞} we need to recall the conformally invariant, $4k$-dimensional, σ-model action introduced in [24], [14]:

$$S_{4k-1} = -\frac{1}{(2k)!} T_{4k-1} \int d^{4k}\sigma \sqrt{h}\, h^{m_1 n_1} \cdot \ldots \cdot h^{m_{2k} n_{2k}} \\ \times \partial_{[\,m_1} X^{\mu_1} \cdot \ldots \cdot \partial_{m_{2k}\,]} X^{\mu_{2k}} \partial_{[\,n_1} X^{\nu_1} \cdot \ldots \cdot \partial_{n_{2k}\,]} X^{\nu_{2k}} \eta_{\mu_1\nu_1} \cdot \ldots \cdot \eta_{\mu_{2k}\nu_{2k}}, \quad (11)$$

where $X^{\mu}(\sigma)$ are the p-brane coordinates in target spacetime; T_{4k-1} is a constant with dimensions of energy per unit $(4k-1)$-volume, or $[T_{4k-1}] = (\text{length})^{-4k}$; $h_{mn}(\sigma)$ is an auxiliary metric tensor providing reparametrization invariant over the p-brane world volume. For the sake of simplicity, we assumed the target spacetime to be flat and contracted the corresponding indices, i.e. $\mu_1, \mu_2, \ldots, \nu_1, \nu_2, \ldots$, by means of a Minkowski tensor. By solving the classical field equation $\delta S_{4k-1}/\delta h_{mn} = 0$ one can write h_{mn} in terms of the induced metric $G_{mn} = \partial_m X^{\mu} \partial_n X_{\mu}$ and recover the Dirac–Nambu–Goto form of the action of (11) and identify T_{4k-1} with the brane tension. If we break the reparametrization invariance of the action (11), by choosing a conformally flat world metric

$$h_{mn} = \exp\left\{2\Omega(\sigma)\right\} \eta_{mn},$$

the resulting, volume preserving diffeomorphism invariant action, is just (9) with a tension given by

$$T_{4k-1} = \frac{1}{g^2_{\text{YM}}} \left(\frac{2\pi}{a}\right)^{4k}. \quad (12)$$

Thus, the large-N limit of the generalized Yang–Mills theory (1) describes a *bag–like*, vacuum domain, or $(4k-1)$-brane, characterized by a tension (12). Being embedded into a $4k$-dimensional target spacetime the bag has no transverse, dynamical, degrees of freedom, i.e. it is a pure volume term. The whole dynamics is confined to the boundary in a way which seems to saturate the holographic principle [25].
We can summarize the results we have obtained in the "flow chart" below.

$$\left|\begin{array}{ccccccccc}
A_{\mu}{}^{i}_{j}\,(x) & \Rightarrow & \boldsymbol{A}_{\mu}{}^{i}_{j} & \mapsto & \mathcal{A}_{\mu}\,(\sigma) & \hookrightarrow & X_{\mu}\,(\sigma) \\
\text{gauge field} & \Rightarrow & \text{matrix} & \mapsto & \text{Weyl symbol} & \hookrightarrow & \text{brane coordinate} \\
S^{\mathrm{GYM}} & \Rightarrow & S^{\mathrm{q,GYM}}_{\mathrm{red.}} & \mapsto & W^{\mathrm{GYM}} & \hookrightarrow & S^{p=4k-1}_{\mathrm{DT}} = \text{volume term} \\
S^{\theta} & \Rightarrow & S^{q,\theta}_{\mathrm{red.}} & \mapsto & W^{\theta} & \hookrightarrow & S^{p=4k-2}_{\mathrm{CS}} = \text{boundary term}
\end{array}\right|$$

The various arrows respectively represent:

$\Rightarrow$ quenching and zero volume limit;

$\mapsto$ Weyl–Wigner–Moyal mapping;

$\hookrightarrow$ large-N limit.

Through all these operations we transformed the generalized Yang–Mills theory, described by (1) and (2) into an *effective theory* for higher dimensional, $(4k-1)$-dimensional, vacuum domain of large-N, generalized Yang–Mills theory, bounded by a $(4k-2)$-dimensional Chern–Simons brane. We conclude this letter with some remarks about a D–brane interpretation of our result. It has been proved that an $SU(\,N\,)$ Yang–Mills theory, dimensionally reduced from $d=10$ to $d=p+1$, encodes the low energy dynamics of a stack of N coincident Dirichlet p–branes [26]. This physical interpretation follows from the matrix, non–commuting, features shared by Yang–Mills fields and D–branes coordinates. This basic feature is encoded into the generalized action (1) as well. We have only to consider that the the specific form of the action (1) is spacetime dimension dependent. However, if we give up conformal invariance in arbitrary number of spacetime dimensions and stick to the $4k$–type form of the Dolan Tchrakian Lagrangian, we can look at the action (1) as the result of a dimensional reduction from $d>4k$ to $4k$. Then, D–brane picture can be applied again. Matching this picture with the end results of our procedure, we conclude that the low energy effective action of a large number of coincident $4k-1$ Dirichlet branes can be approximated the action (9). This conjecture deserves further investigation.

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